Journal of Aircraft

VOLUME 8 MARCH 1971 NUMBER 3

Fatigue and Fracture Mechanics

Herbert F. Hardrath
NASA Langley Research Center, Hampton, Va.

Nomenclature

a = half length of crack, in.
a_c = critical half length of crack, in.
a_0 = initial half length of crack, in.
c = const
E = Young's modulus, ksi
e = nominal strain
g = acceleration due to gravity, 32.2 ft/sec^2
K = stress-intensity factor, ksi-in.$^{1/2}$
K_c = critical stress-intensity factor for failure, ksi-in.$^{1/2}$
K_F = fatigue factor
K_{Ic} = critical stress-intensity factor for plane strain failure, ksi-in.$^{1/2}$
K_{Ii} = initial stress intensity, ksi-in.$^{1/2}$
K_{Iscc} = threshold stress-intensity factor for stress corrosion cracking, ksi-in.$^{1/2}$
K_T = theoretical elastic stress-concentration factor
K_ϵ = strain-concentration factor
K_σ = stress-concentration factor
m = exponent
N = number of cycles required to produce failure at a given stress level
n = number of stress cycles applied at a given stress level
R = ratio of minimum stress to maximum stress
r = radial coordinate, in.
S = nominal stress level, ksi
S_e = fatigue limit, ksi
S_L = limit stress, ksi
S_{max} = maximum stress, ksi
S_u = ultimate strength, ksi
s = standard deviation
Δe = range of nominal strain
ΔK = range of the stress-intensity factor, ksi-in.$^{1/2}$
ΔS = range of nominal stress, ksi
$\Delta\epsilon$ = range of local strain
$\Delta\sigma$ = range of local stress, ksi
ϵ = local strain
ρ = radius of curvature at tip of crack, in.
σ = local stress, ksi
σ_y = local stress in y direction, ksi

Introduction

FATIGUE failures and static failures of flawed structures have occurred in a wide variety of structures for as long as metals have been used. Extensive research has been directed toward understanding the phenomena and toward preventing such failures. Woehler's work in 1858[1] and Griffith's in 1920[2] are frequently cited as the pioneer investigations in fatigue and fracture, respectively. Research in both fields accelerated markedly as the result of widely publicized accidents during and since World War II. Failures of hundreds of welded ships, bridges, storage tanks, and pipelines aroused a strong interest in fracture mechanics,[3,4] and failures in civil airliners about 1947 aroused new interest in combating fatigue in aircraft structures. The unfortunate Comet aircraft disasters in 1954 caused grave concern in both fields and played a strong role in the adoption of the so-called "fail-safe" design philosophy for many transport aircraft.

In preparing this survey of fatigue and fracture mechanics, a review of the several hundred papers that have been published annually in each of these fields during the past two or three decades was clearly impossible. Thus, the scope of the paper is limited to a discussion of behavior of metallic structures and only key concepts are treated. The detailed explanation of even these key concepts is left to cited references and to a selected bibliography, which should help the reader delve deeper if he has the interest and need. Because the paper is written from the point of view of the aircraft designer, considerations on the metallurgical, and atomic levels are not discussed.

This presentation is organized according to the steps that must be taken to design an efficient aircraft structure to operate with minimum danger of fatigue failures in a real environment for some specified life. The first of these steps (preliminary design) is concerned with establishing design requirements and satisfying the static strength criteria that are chosen. The second step is a fatigue analysis with its many component parts. The third step includes fracture

Herbert F. Hardrath is Assistant Chief, Materials Division, NASA Langley Research Center, Hampton, Va. He attended Case-Western Reserve University from which he was awarded B.S. and M.S. degrees in Civil Engineering. He has been employed by NASA since 1947. During this time, he conducted or supervised research on fatigue and fracture of aerospace structures and materials, leading to the publication of over 100 reports. He is currently Chairman of Committee E09 on Fatigue of the American Society for Testing and Materials, the U.S. member of the International Committee on Aeronautical Fatigue, and a member of the U.S. Air Force Aeronautical Systems Division—Industry Advisory Group for the Aircraft Structural Integrity Program.

Presented as Paper 70-512 at the AIAA/ASME 11th Structures, Structural Dynamics, and Materials Conference, Denver, Colo., April 22–24, 1970; submitted May 18, 1970; revision received November 5, 1970. The author gratefully acknowledges the able and willing assistance of his colleagues in the Materials Division, NASA Langley Research Center, especially those in the Structural Integrity Branch, who helped organize the manuscript and made valuable contributions to its content.

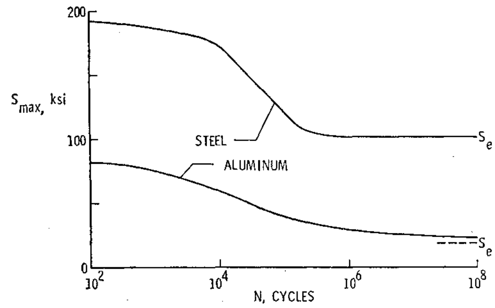

Fig. 1 Fatigue properties of structural materials.

and fatigue crack propagation analyses. In each step many tests are required to check the adequacy of the design, and periodic inspections are required during service to identify and correct damage before catastrophic failure occurs. The frequency and sensitivity with which these inspections must be performed are discussed.

Portions of the discussion are concerned specifically with military aircraft or commercial transport aircraft because the two types of vehicles are subjected to very different service environments and are designed to fundamentally different criteria. The sections on Fracture Mechanics include some applications to missiles and space vehicles.

Preliminary Design

Structural Requirements

When a new aircraft is planned, the manufacturer and customer must agree on a set of requirements. Because engineering technology and manufacturing capability are becoming highly sophisticated, and because a strong competitive urge persists, these requirements usually include higher payloads, lower structural weight fractions, higher speeds, greater ranges, and longer economic lives than were required in previous models.

Frequently, these requirements are so stringent that the designer faces a most difficult challenge to achieve the desired result. He may be forced to abandon some of the design concepts he developed earlier and consider new materials, fabrication processes, fastener design, and the like. Whenever the new ideas have not been proven by experience, unanticipated deficiencies may arise. Furthermore, much structural design is based upon empirical rules and judgments which may not apply to new situations that depart significantly from previous experience. Because of the very large number of factors that must be considered, a new design is accomplished by an iterative process that begins with simple considerations and becomes progressively more refined.

Static Strength

The design and certification of an aircraft structure depends to a major extent on static strength requirements. Generally, the certifying agency (FAA, USAF, or USN) requires that a specified (limit) load shall not cause significant permanent deformation of the structure. In addition, an ultimate load (usually 150% of the limit load) shall not produce failure of the structure. Under very strong motivations to produce structures having minimum weight, designers strive to achieve near-zero margins of safety with respect to these static strength requirements throughout the structure.

Design procedures for satisfying these criteria have been well developed over many years. The basic concepts of material strength and stiffness are ones with which all engineers are familiar. The analytical tools, though highly sophisticated, need to consider only the gross properties of the materials and structural shapes to achieve the desired resistance to buckling, extensive plastic deformation, or failure.

Frequently, at least for military aircraft, a full-scale static test is required to demonstrate that the structure will not fail under any load up to and including the ultimate load. Static tests of civil aircraft are not required beyond limit load. Unless the designer has overlooked some unusually high local stress condition, a redundant structure constructed of a reasonably ductile material will generally pass the required test. The ductility inherent in many materials serves to redistribute local high stresses, thereby reducing their influence on the over-all strength.

Once the demonstrations are completed, the basic static strength problem is no longer a concern because the airframe will probably never experience the ultimate load condition in service, and the variation in strength between successive units is usually small. Because design criteria are based primarily on static strength considerations, most materials are selected, and components are arranged and sized as the result of these considerations. Unfortunately, adequate fatigue and fracture resistance are not assured by satisfactory static strength.

Fatigue Analysis

The achievement of a long economic life for an aircraft structure requires detailed attention to a very large number of local stress conditions and to a large number of operational parameters. Generally, the parameters can be classified into four categories: material behavior; effect of environment, thermal and chemical; structural configuration; service loadings. To a first approximation each of these categories is considered in light of previous experience, and calculations are made by a set of empirical rules to produce an estimate of life. The following sections discuss some of these rules and assess their capabilities and limitations. Because considerable scatter is involved in many of the inputs and responses, each should, in principle, be handled statistically.

Material Behavior

The most common means of characterizing the fatigue behavior of materials subjected to cyclic stresses of constant amplitude is the well-known *S-N* curve (Fig. 1). For many steels, the *S-N* curve exhibits a distinct horizontal portion, called the fatigue limit S_e, below which failure is not expected to occur. For most aluminum alloys the *S-N* curves display a downward slope, even at very long lives. For this reason, an effective fatigue limit is sometimes described at some arbitrarily long life (frequently 5×10^8 cycles).

For many applications in ground-based equipment, operating stresses may be limited to levels below the fatigue limit for steel and below the "effective fatigue limit" for aluminum alloys. However, the high premium on low weight in aeronautical structures precludes such action and operating stresses in aircraft penetrate well into the range where finite lives are experienced. Thus, the sloping part of the *S-N* curve becomes most important.

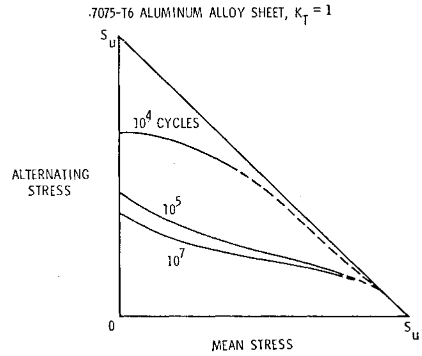

Fig. 2 Alternating stress vs mean stress.

The S-N curves are frequently regarded as characterizing the performance of materials tested. To obtain this performance, test specimens are usually polished very carefully, tests are conducted in carefully alined and calibrated machines, and the test environment is controlled to minimize corrosive effects. Among the parameters that might be investigated by modifying some of the test conditions are the effects of speed of loading, heat-treatment procedures, surface finish, and others. The importance of each of these effects is usually assessed by comparing test results with the characteristic results discussed earlier. Such comparisons are usually empirical. Sometimes a rationale is developed for explaining the observed results, but quantitative rules for anticipating results for conditions not yet tested are usually not available. Attempts have been made to develop analytical expressions for predicting basic fatigue behavior from other properties of materials,[5–9] but for the most part, these expressions have not found application in the design of complex structures.

Many other factors such as the effect of mean stress on life must be considered in a fatigue analysis. Several standard diagrams[10–12] have been employed that show constant-life contours in a space representing the alternating and the mean stress. The diagram in Fig. 2 is typical and was taken from Ref. 11. The diagram is a cross-plot of a family of S-N curves obtained from results of tests in which mean and alternating stresses were systematically varied. This diagram is most useful in fatigue analysis because the analyst is frequently called upon to establish "equivalence" among various combinations of mean and alternating stresses. The curves in the diagram usually indicate that at a given life, higher alternating stresses may be tolerated for a lower mean stress than for a higher mean stress. This trend is exploited in many applications by shot-peening, surface rolling, or other methods of providing compressive residual stresses.

A few investigators[10,13–16] have proposed standard shapes for the curves in the alternating mean stress diagram. However, these standard shapes appear to fit only selected sets of available data. In view of the statistical nature of fatigue behavior, the curves in an alternating mean stress diagram are subject to statistical interpretation, a fact that is frequently overlooked.

Other factors that may affect fatigue behavior are biaxial and triaxial stress states. Numerous investigations[15,17–20] have been conducted to correlate this effect by one of the usual strength theories. However, most of the important fatigue problems confronting designers involve cracks that initiate at edges of free surfaces where a uniaxial stress state is present. Analyses of practical cases where significant biaxial stresses are present are usually based upon the maximum principal stress.

Up to this point the "tools" described are purely empirical. Large numbers of tests are required to establish trends. No deliberate account is taken of the progressive nature of fatigue damage: crack initiation, crack propagation, and final failure.

Effect of Environment

The corrosive effects of the environment in which an aircraft operates can be deleterious to fatigue life.[21–26] Many attempts have been made to evaluate corrosion susceptibility in accelerated tests. At best, such tests can rank various materials for their resistance to corrosion in a specified environment and can demonstrate the effectiveness of protection. Rarely are the results applicable in a quantitative sense to a calculation of life. Usually, the more susceptible materials are avoided or are protected by paints or other special treatments. In service, the structure is inspected for signs of corrosion, parts are replaced if necessary or protective measures are restored.

The effects of temperature have not been significant in most aircraft structures to date, but the design of some of the supersonic military craft and the proposed supersonic transport require that this parameter be given careful consideration.[27] For such aircraft, a new set of materials, principally titanium alloys, will be utilized; new construction methods will be incorporated; thermal stresses will be added to the flight loadings to further complicate the stresses that must be considered; and to the extent that creep might influence behavior, the fatigue life becomes more sensitive to frequency of loading (or total time in test) than is the case at normal temperatures.[28–30]

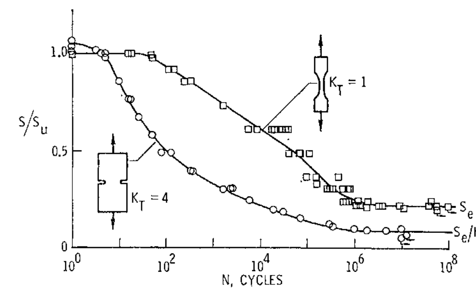

Fig. 3 Effect of stress concentration on fatigue properties of 7075-T6 aluminum alloy.

Structural Configurations

Structural parts inevitably contain changes in shape and mechanically fastened joints which introduce local stress concentrations. Because fatigue failures almost always originate at such discontinuities, much research has been conducted to investigate these effects in a systematic way. Usually, specimen configurations amenable to simple stress analyses are tested and new families of S-N curves are obtained.

The S-N curves shown in Fig. 3 are for 7075-T6 aluminum alloy sheet specimens[31] subjected to completely reversed axial loads. The upper curve is for unnotched specimens and the lower curve for notched specimens. The configuration of the notched specimen is such that elastic stress analysis yields a theoretical stress concentration factor K_T of 4.

The simplest and most common analysis of these results is the computation of the ratio of the fatigue strengths at a given life. This ratio is commonly called the "fatigue factor" and is represented by the symbol K_F. In this paper, K_F is applied only to cases of completely reversed stress and at long lives. At such long lives, K_F is usually smaller than K_T. The magnitude of the difference depends upon the material and the size of part. Empirical rules have been developed[32–34] which help to predict K_F for a given case, but these will not be discussed here. Suffice it to say that K_F is smallest for small specimens. This fact alone warns against the sometimes-recommended test of a scaled-down structure or component because such tests would inevitably produce longer lives than would be found in full-scale tests.

At shorter lives the ratio of strengths is less than K_F and, for failure during the first cycle, it is reasonably close to one for ductile materials. This trend is attributable to plastic action near the stress raiser, a phenomenon that has been studied by a number of investigators.[35–43] The behavior is illustrated schematically in the right-hand portion of Fig. 4. The sketch shows the stress distribution across the net section of the notched plate subjected to an axial stress S. If the local stress is elastic, the distribution represented by the dashed curve applies and the maximum stress is K_TS as represented by the square symbol. However, when the local stress exceeds the yield strength, the stress is redistributed as shown by the solid curve and the maximum local stress is $K_\sigma S$ as indicated by the circular symbol. Neuber[44] derived the currently favored expression for computing the stress concentration factor K_σ in a notched specimen sub-

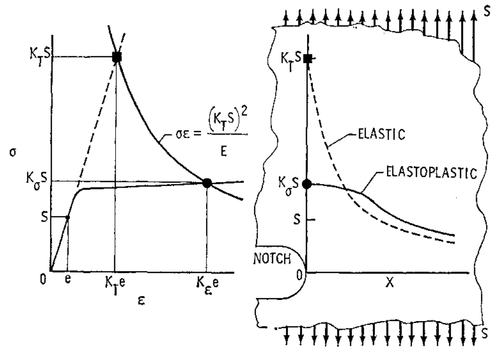

Fig. 4 Effect of plasticity on stress concentrations.

jected to a static load: $K_\sigma K_\epsilon = K_T^2 = (\sigma/S)\epsilon/e$ where K_σ and K_ϵ are the stress concentration factor and strain concentration factor, respectively, σ and ϵ are the local stress and strain, respectively, and S and e are the nominal stress and strain, respectively. For elastic nominal strains, this equation may be written $\sigma\epsilon = (K_TS)^2/E$, where E is Young's modulus. This equation is shown in Fig. 4 along with a graphical representation of its solution for a given case. The heavy line represents the stress-strain curve for the material and the dashed line is the extension of the elastic portion of the curve to the local stress K_TS, which would occur if plastic action were not present. The remaining curve is the locus of points satisfying the Neuber equation for all cases wherein the values K_TS are equal to the one shown. The intersection of this curve with the stress-strain curve represents the local stress and strain condition for the case in question.

Based on the Neuber analysis, Morrow and his associates[45] have chosen the parameter $(\Delta\sigma\Delta\epsilon E)^{1/2} = K_F(\Delta S\Delta eE)^{1/2}$, to correlate fatigue behavior of notched and unnotched specimens. In this parameter, K_F is substituted for K_T in the Neuber equation to account for size effect, and the symbol Δ denotes the range of the stress or strain. The cyclic stress-strain curve is substituted for the static stress-strain curve in Fig. 4 to determine the stabilized local stresses and strains under cyclic loading. When the data shown previously in Fig. 3 are replotted on the basis of this parameter, the correlation seen in Fig. 5 is achieved. The data points for both the unnotched and notched specimens have converged into a single band.

This correlation has been achieved by employing rational notions, namely, the stress concentration factor for the plastic range and the cyclic stress-strain curve. However, the system has not yet been developed adequately for loadings other than completely reversed nor for complex configurations.

Obviously, the designer must deal with complicated configurations including joints. A considerable number of systematic investigations have been conducted to study fatigue behavior of simple joints[46–51] and of complete aircraft structures.[52–60] Usually, the results of such investigations are most useful for the vehicle tested. In some cases, they may identify trends that are informative in a qualitative sense but

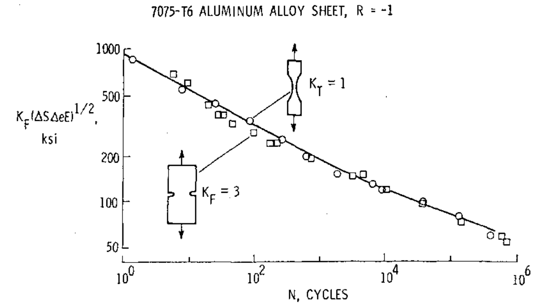

Fig. 5 Correlation of fatigue behavior for notched and unnotched specimens.

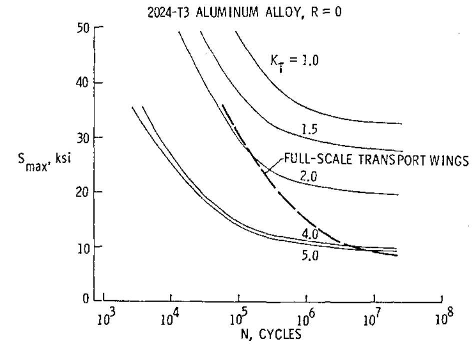

Fig. 6 Comparison of fatigue behavior for structural components and notched specimens.

do not provide quantitative information that will allow a designer to estimate a life for a design somewhat different from that tested. Thus, quite frequently, fatigue tests are conducted on complex components representing some part of an airframe structure to determine the "quality of construction" achieved in that component. For convenience, results are plotted on diagrams containing S-N curves for simple specimens (Fig. 6) to find an "effective K_T," an index that might be useful for comparisons of fatigue adequacy for various designs. The solid curves in Fig. 6 are S-N curves[61–64] for several sets of specimens with notches of various severities, all tested under cyclic tension stresses. The dashed curve is for results from fatigue tests of 16 transport aircraft wings.[53] The "effective K_T" determined in this manner has widely different values depending upon the stress level of the full-scale test, and could lead to misleading assessments of the adequacy of a given configuration.

This disconcerting result is probably obtained because the component contains many structural features each of which has its own K_T. Because each of these features is subjected to a slightly different nominal stress and perhaps a different local residual stress, the behavior of the component should be considered to be represented by the conceptual curves of Fig. 7. The several solid curves each represent a separate local feature with its own K_T. The stress level for each curve is adjusted to account for the local nominal stress and residual stress state. Tests of the complete component are likely to result in failures along the dashed curve, which is the lower envelope for the solid curves and the mode of failure is quite likely to be different, depending upon the stress level of a given test. Unusually good details might have basic S-N curves like the one labeled A in Fig. 7. Failure at such a station is highly unlikely. Conversely, a very poor detail would be characterized by a curve far below and to the left of the others and might well govern the behavior of the structure of many stress levels.

While the foregoing is a rational conceptual model of why structural tests behave as they do, it is hardly helpful as a design tool unless one could predict the relative position of all the local S-N curves and then compute the envelope. This envelope cannot be computed by current technology. Thus, one is limited to identifying some worst cases and to

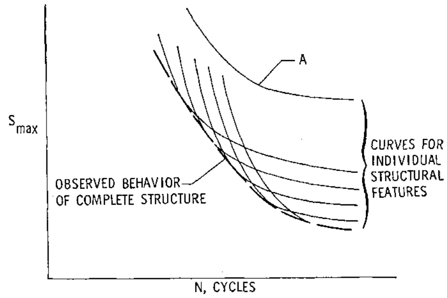

Fig. 7 Fatigue behavior of structural components.

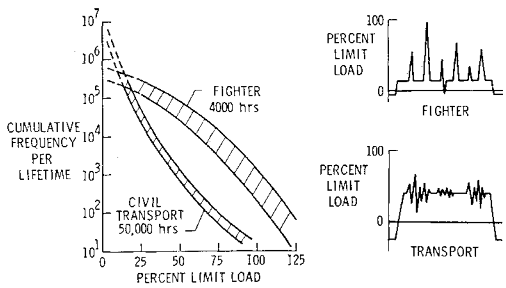

Fig. 8 Load experience in aircraft.

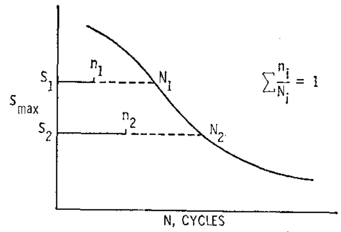

Fig. 9 Miner's rule.

indicate trends.[65] Qualitative reasoning and intuitive judgment[66] play as large a role as formal calculation. A very large number of components and complete air frames are tested to back up these judgments.

Service Loading

The foregoing discussion is limited to constant amplitude loadings, but service loadings are almost always much more complex and arise from a variety of sources such as gusts, maneuvers, landings, acoustic excitation, thermal gradients, and pressurization. Only the primary structural loads are considered in this discussion. For a given aircraft type, the expected load experience is either derived for a specified mission or prescribed contractually. Gust and maneuver loading statistics have been collected for many years by NASA[67–69] and the loading statistics for military vehicles are specified in the appropriate military handbooks.[70–71] The required life, the frequency of occurrence of a given load, and a representative stress history for a flight of a transport and a fighter are shown in Fig. 8. The loading spectra shown are for accelerations at the center of gravity of the aircraft. Stations on the wing or fuselage at which strains correlate closely with such accelerations will, of course, experience stress spectra proportional to the curves shown. Other stations will have stress spectra very different from these. Several observations are pertinent. The load spectrum for the transport is subject to considerably less scatter than that of a fighter. The transport rarely, if ever, experiences limit load while the fighter encounters loads considerably in excess of limit quite frequently. The transport, which is expected to be serviceable for many more hours than the fighter, experiences many times more low load cycles than does the fighter. The representative stress histories indicate that the gust-sensitive transport experiences loading cycles that are reasonably symmetrical about the 1-g stress level. The fighter loads come mostly from maneuvers and they are mostly on one side of the 1-g stress level. For wing stations outboard of the landing gear, the transport experiences an important ground-air-ground cycle. This cycle is usually less important in fighters. Helicopters characteristically experience very large number of stress cycles and relatively fewer high load cycles. For the rotor blades, the cyclic stresses are superposed on a moderately high steady stress produced by centrifugal forces. These considerations plus many others lead to rather different approaches to the design of these vehicles.

The estimation of a service life under these loadings involves some type of "damage" analysis. The well-known Miner[72] hypothesis, described in Fig. 9, is the most commonly used. Some authors refer to this hypothesis as the Palmgren-Miner rule to emphasize the fact that the method was first proposed by Palmgren[73] some 20 years earlier. The rule simply states that failure will occur when $\Sigma n_i/N_i = 1$ where n_i is the number of cycles of S_i and N_i is the number of cycles of stress required to cause failure at S_i. This is probably the simplest, most straightforward rule possible. Miner qualified the application of this rule to predict the initiation of a crack, but this qualification is usually ignored and the rule is applied for a variety of failure criteria.

Several other cumulative damage rules have been proposed by others[8,74–83] but none have gained wide acceptance. Usually, they are more complicated, but not necessarily more accurate when their predictions are compared against data taken from a large number of papers in the literature.

The application of a damage rule like Miner's requires the S-N curve for constant-amplitude tests of the same configuration and for the same combinations of mean and alternating stress as those for which the calculation is desired. This information is rarely, if ever, available in sufficient detail, particularly for major components. Consequently, designers frequently perform calculations of $\Sigma n/N$ using S-N data for simpler specimens taken from the literature. Sometimes, the "quality of construction," discussed in an earlier section, is evaluated in a programed-load test. The result is analyzed by the $\Sigma n/N$ method using sets of S-N curves representing various K_T values and the calculation that comes closest to producing $\Sigma n/N = 1$ identifies the effective K_T for the configuration. This approach is as likely to lead to questionable assessments as did the approach based on constant-amplitude stress conditions previously discussed.

Miner's rule usually fails to predict the life at failure quantitatively. Many reasons may be cited for this discrepancy. Systematic series of tests[84–90] have shown that $\Sigma n/N$ varies with the sequence in which loadings are applied, with the over-all mean stress, and with whether or not negative loadings are included as indicated in Fig. 10. These results suggest that an empirically adjusted value of $\Sigma n/N$ might produce better estimates of life than does the standard value of one. The trends outlined in Fig. 10 have been corroborated to a limited extent in other tests on simple structural components.[91]

The variation in $\Sigma n/N$ as a function of the parameters cited can be explained by considering the local stress near a hole or discontinuity in the structure. Because of the stress concentration present, the local stress frequently exceeds the yield strength of the material when the applied load is moderately high. The local behavior is indicated in Fig. 11. A high tensile applied load causes the local stress to reach point A. Subsequent unloading produces a local stress change along the line A-B, and the compression residual stress B is left when the applied load is completely removed. The local stress due to subsequent load must be adjusted to account for this residual stress. Unless the subsequent load is high enough to again produce local plastic deformations, the amplitude of local stress will be unchanged from that cal-

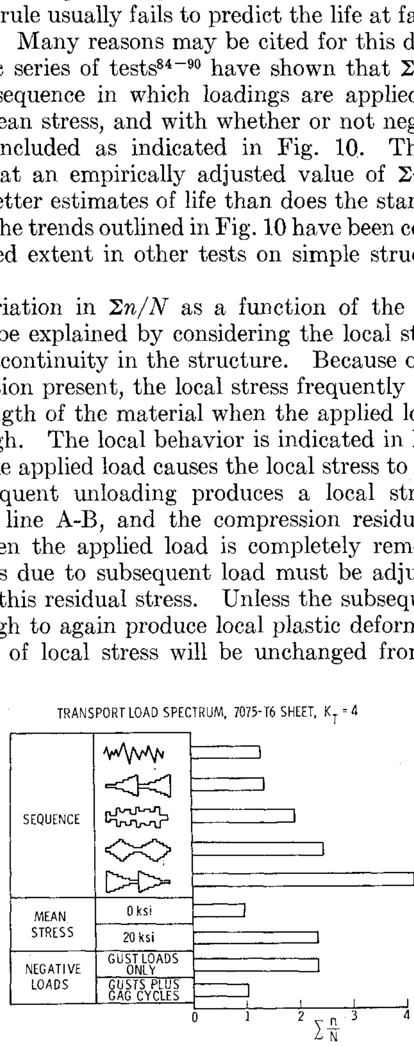

Fig. 10 Loading factors influencing $\Sigma n/N$.

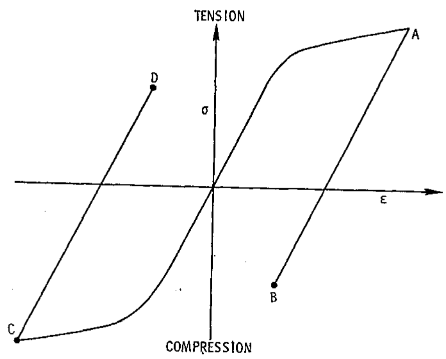

Fig. 11 Local residual stress.

culated from elastic considerations. However, the mean local stress is reduced by the residual stress present and, as seen earlier in Fig. 2, a longer life is to be expected than would be observed for the same range of stress, but with a higher mean stress. Conversely, a large compressive load produces a local compressive stress C and leaves a residual stress D upon unloading. Consequently, shorter lives are expected.

These effects are introduced each time a high load is applied during a fatigue test or in service. For the situations portrayed in Fig. 10, compressive residual stresses are more effective in prolonging life for monotonically decreasing sequences of stress amplitudes than for other sequences. Similarly, they are more effective for positive mean stress than for zero mean stress because high compressive loads (and corresponding tensile residual stresses) occur very infrequently.

The fact that $\Sigma n/N > 1$ in many cases results in somewhat longer lives than are calculated. Generally, no deliberate advantage is taken of this added life; thus, some conservatism is introduced. However, the fact that $\Sigma n/N$ may be significantly less than one in some cases is frequently not recognized; thus, the actual life may be shorter than estimated by $\Sigma n/N = 1$.

Although the observations just discussed provide a means for understanding some of the apparent anomalies observed in fatigue behavior under variable-amplitude loading, design tools based upon them are not yet adequate to predict life without extensive testing. Because fatigue life must be calculated for each of a huge number of stations in an airframe, each with its own local stress problems, the task becomes gargantuan in size and its accuracy is questionable. Add to this the inherent scatter, which will be discussed in a subsequent section, and one almost despairs of ever accomplishing the task in the formal sense described.

This state of affairs dictates that the best estimate of expected life is to be gained from a fatigue test of a complete airframe under programed loads simulating those expected in service. Such a test has been conducted on a great many airframes during the past several decades. The design of this test should, of course, consider some of the systematic influences cited in this section. For example, loadings should be applied in random or flight-by-flight sequence, if possible. Economic factors frequently prevent this course of action, but the systematic influence of other sequences should be recognized. Economic considerations sometimes suggest that negative loadings be deleted—an action which ignores one of the predominant damage-producing loadings experienced by aircraft.

An inappropriately designed test may well introduce systematic errors in the estimated life, many of them in the optimistic direction. Thus, the test of a complete aircraft does more toward identifying the weak details in a structure and evaluating fixes than it does toward establishing a "safe life." Further, a full-scale test may not identify the appropriate critical weak details because of the inherent limitations of the methods used and because some significant environmental conditions may not be simulated adequately in the test.

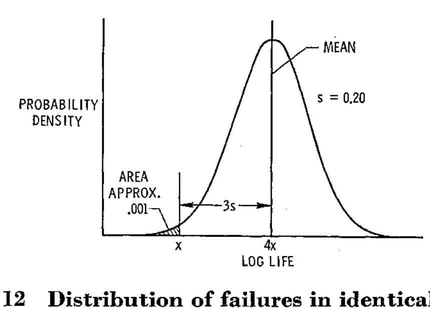

Fig. 12 Distribution of failures in identical tests.

Scatter in Fatigue

The fact that considerable scatter is present in fatigue test results has been mentioned in previous sections of this paper. Similarly, the loadings to which aircraft are subjected are characterized by considerable variability. Thus, the results of a fatigue test of a complete airframe must be interpreted in light of the statistical nature of the basic inputs and responses.

Few truly satisfying statistical studies have been performed to define adequately the statistical parameters needed to anticipate the scatter to be expected in a given situation. Similarly, customers and regulatory agencies have not specified the level of reliability they require. Several years ago, Lundberg[92–94] recommended a statistical approach to the over-all fatigue design problem. At the time, his proposal was felt by many to require unnecessarily severe criteria. Such methods have not been adopted formally to date. However, increasing emphasis is being placed on the assessment of structural integrity and life in terms of reliability concepts. Thus, it is of interest to comment briefly here.

Without formal and extensive statistical treatments, a widespread belief has developed that fatigue tests of structural components display less scatter than do tests of simple specimens, and programed load tests display less scatter than constant amplitude tests. By far, the most extensive systematic series of fatigue tests of an aircraft type was conducted in Australia[56] on ninety-two P-51 aircraft. The standard deviation of the log of the life was found to be about 0.20. That result was interpreted to require that the life from a single test be divided by a "scatter factor" of 7.5 to preclude more than one failure in a fleet of 1000 aircraft. The present practice of the U.S. Air Force is to require a fatigue test of a complete airframe to survive four times the number of cycles expected during the life of that aircraft type. An anticipated spectrum of loads having a representative severity is employed. The Navy requires a test to twice the expected life, but the spectrum is admittedly more severe than expected. Recent studies based on early-order statistical techniques[95–96] suggest that a factor of more nearly 8 is needed to relate the earliest failure in a large fleet of aircraft to the mean life.

A simple statistical example may help to illustrate the underlying assumptions and probable significance of the foregoing observations. The diagram in Fig. 12 is the familiar normal distribution curve as applied to results of a fatigue test of a complete airframe. The "safe" life is denoted on the abscissa by X and the test life required by Air Force specifications by $4X$. The probability density of failure at a given life is plotted as the ordinate. The implicit assumption is that the life to failure in the test establishes the mean life for that type of failure in the fleet. The figure is drawn for a standard deviation s of the log life equal to 0.20. For that value of s the life X coincides with the mean-minus-$3s$ life. The shaded area to the left of X represents the probability of failure occurring before X; in this case, approximately 0.001. In other words, 1 in a fleet of 1000 aircraft would be expected to experience the failure in question.

Several comments are needed to place this diagram in perspective. The result of one test *does not establish a mean.* The statistical diagram represents identical tests of nominally identical specimens, conditions which are far from realized in service. The sequence of loading, inclusion or exclusion of negative loads, and the service environment were cited earlier as being significant but are rarely simulated adequately. To the extent that any of these and the airframe tested are not representative of service conditions, the estimated service life may be seriously in error. For example, a photograph of a detail that caused a fatigue failure during a fatigue test of a large aircraft is shown in Fig. 13. The crack originated at a very shallow machining mark left on an otherwise-generous fillet in the corner of an access opening. Flaws of this type are probably not typical of the aircraft type. However, had it been present in a service aircraft rather than the one in test, it could have caused failure, but would not have been found in test. Thus, it is virtually impossible to predict a safe life with satisfactory confidence even though a complete airframe was tested.

The realization of this fact has led the manufacturers of commerical transport aircraft and the FAA to place much more emphasis on fail-safe construction and on inspections and maintenance to assure reliability in service. This action is taken in spite of the fact that the service loadings of transports are under much closer control than for other types of aircraft, and in spite of the economic need to operate many of these aircraft for over 12 hours of every day.

The uncertainties in anticipating service loadings, the inability to identify unrepresentative—but potentially dangerous—details by tests, and the inadequacy of analytical procedures indicate that the best index of the life of a structure that is presumed to have a "safe life" is behavior in service. Fortunately, many structures develop inspectable cracks in advance of catastrophic failure. These cracks can only be detected by periodic and careful inspections. Contrary to common opinion, such inspections are even more vital in safe-life structures that are not protected by fail-safe backups. Guidelines for such inspections are developed in the succeeding sections dealing with Fracture Mechanics.

Fracture Mechanics

The fail-safe design philosophy depends strongly on consideration of residual strength of partially failed structures and of rates of crack propagation—the key features of the fatigue process that are all but ignored in the conventional safe-life calculations and verifications.

A rationale for the analysis of fracture has been developed based on the work of Griffith and Irwin[2,97] and has come to be known as "Fracture Mechanics." A special committee (E-24) was established within the American Society for Testing and Materials to deal with this class of problems. Because of the concerted efforts of this group, Fracture Mechanics has been developed in a much more rigorous way than fatigue analysis. However, the present methodology does not yet handle adequately many of the practical fracture problems confronting designers. A large number of competing methods for analysis of fracture have been developed and are used in a number of companies and research laboratories.[98–104] For brevity, these latter methods are not discussed in detail. However, many of them deserve study for the solution of particular practical problems.

Basis for Analysis

The fundamental basis for Fracture Mechanics[105–107] is the consideration of a quantity termed "stress intensity." To engineers not trained in the concept of stress intensity, it seems perplexing at first exposure. The sketch and notes in Fig. 14 illustrate the simplest concepts. The sketch shows the elastic stress distribution along a line in the path of a crack in an infinite sheet subjected to a uniformly distributed stress

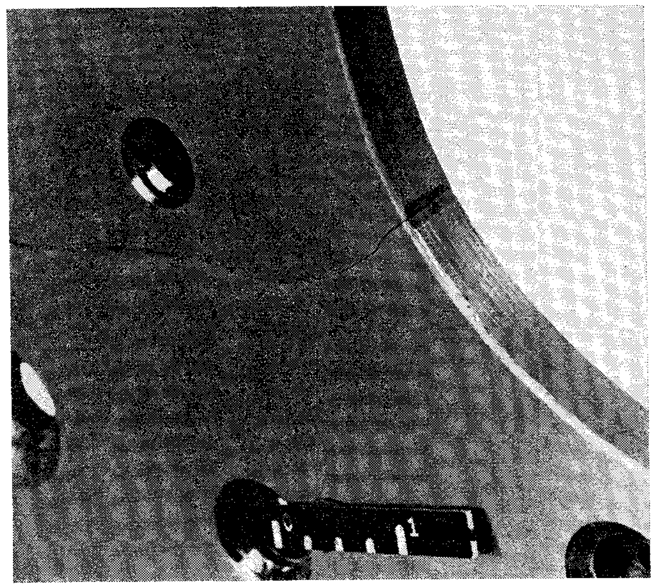

Fig. 13 Fatigue crack at tool mark.

S. Because the crack tip is infinitely sharp, the calculated local stress distribution contains a singularity. The distribution of local stress is given by the equation: $\sigma_y = S + S(\pi a/2\pi r)^{1/2} + \ldots$, where a is the half length of the crack and r is the radial coordinate of any point in the sheet. The numerator of the second term of the equation measures the strength of the $S(\pi a)^{1/2}$ singularity and is called K, the stress intensity factor. [Some authors prefer $k = S(a)^{1/2}$.] It has the somewhat unusual units of ksi (in.)$^{1/2}$. At least one author[108] avoids these units by considering the entire second term of the equation at the point $r = 1/2\ \pi$in., thus, his K is numerically equal to the K used herein, but has the units of stress. Roman numeral subscripts I, II, and III are used to denote three modes of loading: opening mode (shown), in-plane shear, and out-of-plane shear, respectively. All other stress states are derived by appropriate superposition of solutions for these three modes.

If the stress S on the part is increased until the crack extends uncontrollably, the crack is said to have become unstable. The value of K computed for the stress and crack length present at the instant of instability is called the critical value K_c (sometimes called fracture toughness). The analysis is based upon an elastic stress analysis. Thus, the method applies only to very brittle materials or to specimens in which plane strain conditions are present at the crack tip. If these conditions are satisfied, a minimum value of K_c is obtained which is called the plane-strain fracture toughness and is labeled K_{Ic}. A set of empirical criteria have been developed and special specimens have been designed[109] to assure that "valid" values of K_{Ic} are obtained for a given material at minimum cost. Obviously, many practical materials and configurations do not satisfy the restrictive conditions imposed for these tests.

Originally, fracture was expected when the rate of release of energy due to local redistribution of stress balanced the

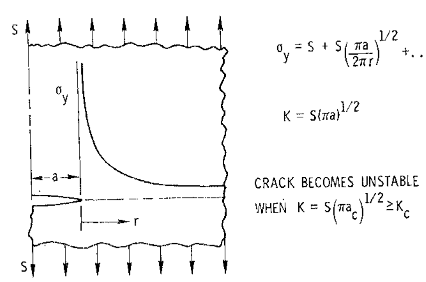

Fig. 14 Stress intensity analysis.

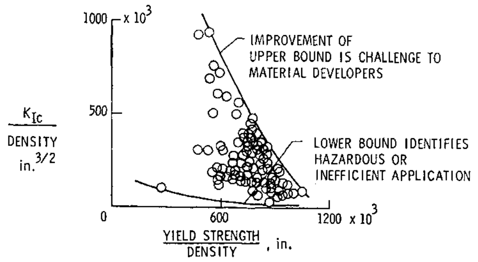

Fig. 15 Correlation of toughness with yield strength for steels and aluminum alloys.

energy required to produce new surfaces as the crack grows. However, attempts to quantify this hypothesis were unsuccessful because the two energies were found to be different by several orders of magnitude. Much of the discrepancy is attributable to plastic work done during the fracture of most engineering materials. Consequently, the value of K_{Ic} is established empirically from test data and is generally considered to be a characteristic property of the material.

A large number of analysts are active in developing mathematical schemes for extending the applicability of the method to a wider range of problems. Among the features considered are the effects of plasticity,[110–117] the finite width of specimen,[118] cracks growing from holes,[119] or cracks growing under special loading conditions.[120–123] At least two new technical journals have been established to publish results of these studies[124–125] and several symposia have been held to discuss results.[126–130]

This activity on the study of fracture has developed several key issues that structural designers should consider, at least in a qualitative sense to guide choice of materials and to plan inspection procedures.

Trends in Results

One significant observation is illustrated in Fig. 15.[131] In this figure, fracture toughness is plotted against the yield strength (both normalized by density) for a variety of structural steels and aluminum alloys. Although the data exhibit considerable variability, they also show an unmistakable trend toward lower toughness for higher yield strength. Generally, this trend is also observed for a given material heat-treated to several strength levels. This single observation explains why many structural parts made of high-strength steels sometimes encounter unexpected failures in service. Materials having K_{Ic} values lying near the lower limits of the scatter shown are likely to lead to hazardous applications unless service stresses are kept low. If design stresses are kept low, the part will weigh considerably more than if a material with a better combination of properties were chosen. Metallurgists and processing engineers are challenged to produce materials with characteristics equaling or exceeding the upper limits of the data shown. Notable among these efforts have been studies of the effect of controlling more closely the percentage of certain alloying constituents in a material[132–133] and the effects of thermomechanical processing.[134]

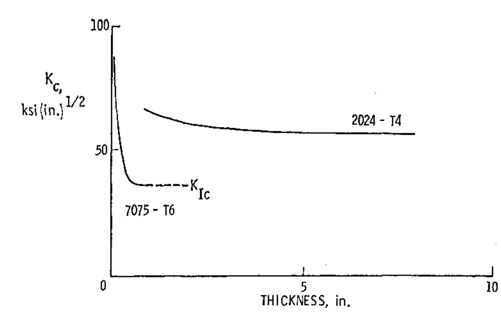

Fig. 16 Effect of thickness on fracture toughness.

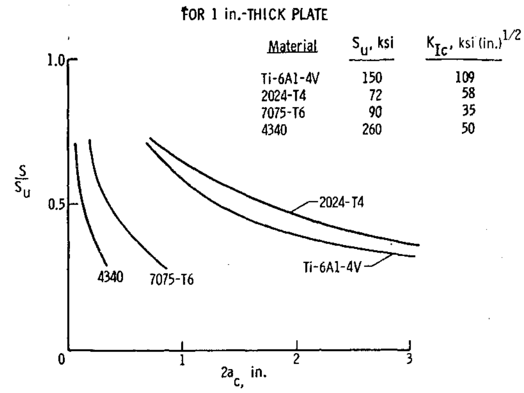

Fig. 17 Critical crack lengths.

Specialists in fracture studies have long appreciated that the thickness or the specimen or part has a profound effect on behavior and on strength. Two typical sets of results[135] are shown in Fig. 16. The toughness data for 2024-T4 and 7075-T6 aluminum alloys are plotted against thickness. The curves show that low values of K_c are achieved in thick specimens. As indicated previously, this behavior is generally attributed to the fact that the material at the crack tip is subjected to plane strain and the minimum value is labeled K_{Ic}. For thinner specimens, the stress state is more nearly plane stress and because significant plastic action takes place, higher toughness values are observed. For a tough material like 2024-T4, a specimen several inches thick must be tested to achieve a valid K_{Ic}.

The Fracture Mechanics method has not yet been extended to treat with confidence the problems corresponding to the sloping or higher portions of the curves in Fig. 16. Most emphasis has been placed on determining the asymptotic lower limit. Thus, the method is not useful for treating a wide range of practical problems unless the fracture data are collected in tests of specimens with representative thicknesses, widths, and other key features. In the latter case, the toughness number, though not a "valid" value of K_{Ic} is, nevertheless, a convenient analytical tool for interpolation and moderate extrapolations. Obviously, the fracture problem in the very tough materials is less serious because a given crack does not reduce residual strength as much proportionately as for less tough materials. If welding is employed in the structure order consideration, the fracture tests should, of course, include specimens containing representative welds.

The curves of Fig. 17 illustrate the importance of fracture characteristics of commonly used materials. The ordinate of the diagram is the applied stress, normalized to the tensile strength of the material, and the abscissa is the crack length for through cracks in wide plates 1-in. thick. The curves represent the residual strength of such plates made of each of four commonly used structural materials. Clearly, for some fixed crack length, the residual strength of the steel has been reduced much more drastically than has that of the tougher materials. Conversely, given a stress level, the rough material can tolerate a much larger crack without causing failure

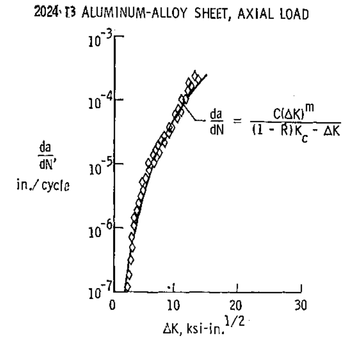

Fig. 18 Fatigue crack propagation.

than can the more brittle material. In service, structural parts made of 2024-T4 aluminum alloy occasionally contain cracks several inches long without failure, but steel parts frequently fracture from very small cracks. Obviously, the inspections required to preclude fracture must be much more sensitive and frequent for high-strength steel parts.

Fatigue Crack Propagation

Although Fracture Mechanics was not developed for use in the analysis and prediction of fatigue crack propagation, the stress intensity parameter has been adapted for the analysis of such data. Generally, test results are plotted, as shown in Fig. 18, as the increment of crack growth per cycle against the range of the stress intensity ΔK. A number of "laws" have been proposed of anticipating the shape of the resultant curve.[136–147] One by Forman [146] has been found on an empirical basis to treat an assortment of data gathered for a variety of combinations of alternating and mean stress and over a wide range of ΔK and crack growth rates.

As in most fatigue analyses, this study has started with constant-amplitude loading. A few studies[148–156] of fatigue crack propagation under variable-amplitude loading have shown that significant delays in crack growth may be observed when the stress level is reduced during a test.

Proof Testing

An important application[157] of residual strength and crack propagation information to a design problem has been in assuring adequate life in pressure vessels employed in space vehicles. Because the expected life is short and weight savings are even more important than in aircraft, high strength materials are used at high stress levels. To prove that the vessels are free of significant flaws, a proof test is conducted on each vessel. As indicated in Fig. 19, the proof stress level is somewhat higher than the operating stress level. Provided the vessel survives the proof test, the basic Fracture Mechanics criterion $K_{Ic} = S(\pi a)^{1/2}$ is used to determine the maximum-sized crack that could have been present without causing failure. That crack length and the operating stress level are used to determine the crack propagation rate and the number of cycles of load required to propagate the crack to its critical length. If that number of cycles is sufficient to allow the necessary pressure cycling during use, the vessel is declared safe. The procedure may, of course, be reversed to determine the relation between proof stress and operating stress that will assure sufficient life.

The application of this rationale to proof tests of aircraft structures presents a more difficult problem. Generally, the aircraft structure is many orders of magnitude more complicated, the numbers of load cycles desired are very much greater, and variable-amplitude loadings are almost always involved. Further, a proof test of a structure like a wing would require that a high bending load be applied. The side of the wing that is loaded in compression might well be

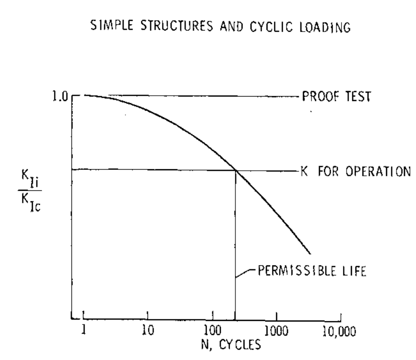

Fig. 19 Proof test for fatigue crack propagation.

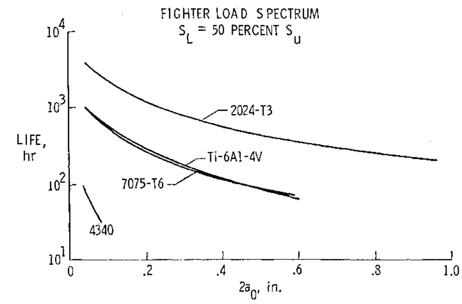

Fig. 20 Life to propagate a crack to failure.

left with tensile residual stresses that could aggravate the fatigue problem. Proof testing is not a demonstrated or accepted technique for determining a safe life for an aircraft structure. Limited studies indicate that practical proof test stress levels assure only a very short useful life after the test. The following discussion of inspection intervals provides additional information to support this conclusion.

Prediction of Inspection Intervals

Structures must be inspected frequently enough so that a crack that is not detected in one inspection will not grow to a length that will cause failure under normal operating conditions within the inspection interval. A prudent criterion is one that requires inspections at one-quarter the calculated period to grow a crack from the minimum inspectable to critical length. This procedure would protect against an occasional crack that is missed during an inspection and allows for the statistical scatter in results.

The curves in Fig. 20 have been calculated to indicate the influence of such a practice on inspections of fighter aircraft. The abscissa is the size of flaw that is presumed to be identifiable in an inspection. The ordinate is the expected life of a part that contains a flaw and that is loaded according to the spectrum of loads for a fighter given in the military specification[70] for fighters. The calculations were made by the Forman expression (Fig. 18) assuming that each material was stressed to 50% of its ultimate tensile strength at limit load. The highest applied load was assumed to be 125% of limit as given in the military specification. Results are shown for the same four materials chosen for the residual strength curves shown in Fig. 17. Actual lives are expected to be somewhat longer than those shown because of favorable interactions produced when stress levels are changed.[152–155] However, the positions of the four curves relative to each other are not expected to change significantly.

The results illustrate clearly that the tough aluminum alloy has a much longer residual life for a given crack length than do any of the other materials. Therefore, a much cruder or less frequent inspection plan would produce a given level of safety. The high strength steel has a very short residual life even when very short cracks are identified. Any useful inspection interval would require that inspections be capable of finding microscopic flaws. The relationship between crack length and residual life will, of course, be correspondingly different for structures stressed to other percentages of the tensile strengths of the materials from which they are made.

The usefulness of a proof test in detecting flaws and assuring a safe life or inspection interval can be indicated by a combined analysis of Figs. 17 and 20. A proof test to a very high load level, such as 70% of the ultimate strength, indicates a safe life of 100–1000 hr, depending on the material. The life, however, is shortest for the high-strength steel which is most sensitive to the small flaws that are hardest to detect by nondestructive evaluation techniques. Note, also, that fracture toughness values alone are not a measure of either the relative strength of cracked parts or the appropriate interval for inspections.

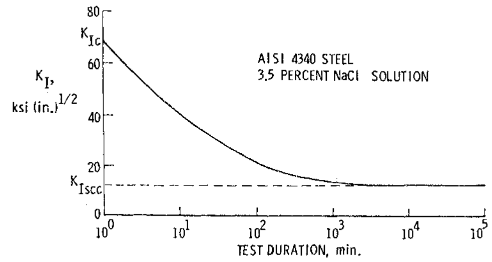

Fig. 21 Stress corrosion cracking.

Stress Corrosion Cracking

The stress intensity factor and basic fracture concepts are also applied in the study of stress corrosion cracking of parts containing cracks (Fig. 21).[158] If a cracked specimen is subjected to a tensile stress for a long time while it is immersed in a corrosive environment, slow crack growth is likely to occur at a K level substantially below K_{Ic}. When the crack grows to the critical length for the applied stress level, fracture will occur. The lowest K value at which this phenomenon occurs is called the stress corrosion threshold and is labeled K_{Iscc}.

This basic concept is of paramount importance in the evaluation of pressure vessels for spacecraft, which may be subjected to highly aggressive environments for long times in space flights. A proof-test procedure for guarding against stress corrosion in pressure vessels has been developed[159] and is illustrated in Fig. 22. The curves represent the locus of all combinations of applied stress and crack length for the values of K_{Ic} and K_{Iscc} taken from Fig. 21. A proof pressure is applied to the vessel and the curve of K_{Ic} indicates that no crack greater than a_0 is present. The safe operating stress for that crack is taken from the curve of K_{Iscc}.

In aircraft, stress corrosion cracking may take place due to moisture from rain, condensation, splashing from runways, and others. Exhaust fumes, salts, and other corrodents may also be present. The behavior is almost always concurrent with crack propagation due to fatigue. Thus, analysis would require that both mechanisms be considered simultaneously.

Prediction of Fatigue Life

A few investigators[160–161] have applied fatigue propagation analysis and Fracture Mechanics to compute fatigue life. A minute flaw (usually a few $\frac{1}{1000}$ in. long) is assumed to exist in any manufactured part. This flaw then propagates under the applied loadings until a critical combination of crack length and applied stress produces failure. A consistent application of the idea would require accounting for the stress field around the discontinuity that initiates failure.[111] Calculations based on these assumptions have given good predictions of observed behavior. However, this proposal has not been established well enough to be recommended as a design tool as of this writing.

Concluding Remarks

The development of an efficient airframe capable of operating economically and safely for a long life poses very complex design problems. Existing design procedures for fatigue are based primarily on empirical rules and results taken from systematic test series. Large numbers of tests are being performed to evaluate the adequacy of designs and much judgment is employed to produce flightworthy structures.

Systematic research is providing ways for rationalizing observed behavior and for anticipating behavior. However, a purely analytical fatigue design method is not yet available. If it were, tremendous numbers of calculations would be required. The scatter inherent in fatigue behavior and in service conditions would require that results of such analyses be interpreted statistically.

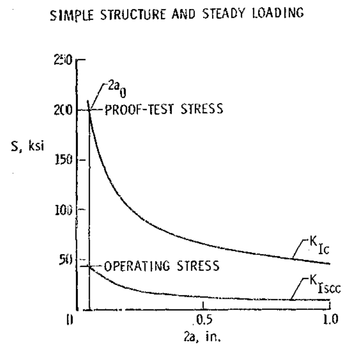

Fig. 22 Proof test for stress corrosion cracking.

Consequently, effective designs will continue to be produced through a judicious combination of experience, intuition, fatigue analysis, and evaluation testing. Fail-safe features should be incorporated whenever practical to prevent catastrophic failures. Frequent and careful inspections are required to maintain continued structural integrity in service.

The use of high-strength materials has increased the chances of major failures caused by moderate damage or flaws. Fracture Mechanics analysis procedures appear capable of treating simple problems of this sort, but are not yet adequate for complex structural and loading conditions. However, simple applications of these procedures provide a useful rationale for defining inspection techniques and schedules.

References

1 Woehler, A., "Über die Festigkeitsversuche mit Eisen und Stahl," *Zeitschrift für Bauwesen*, Vol. 8, 1958; also *English abstract: Engineering* (London), Vol. 11, 1871.

2 Griffith, A. A., "The Phenomenon of Rupture and Flow in Solids," *Philosophical Transactions of the Royal Society of London*, Ser. A, Vol. 221, 1920, pp. 163–198.

3 Parker, E. R., *Brittle Behavior of Engineering Structures*, Wiley, New York, 1957.

4 Boyd, G. M., "Fracture Design Practices for Ship Structures," *Fracture—An Advanced Treatise*, Vol. V, Academic Press, New York, 1969.

5 Coffin, L. F., Jr., "A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal," *Transactions of the American Society of Mechanical Engineers*, Vol. 76, 1954, pp. 931–950.

6 Manson, S. S., *Thermal Stress and Low-Cycle Fatigue*, McGraw-Hill, New York, 1966.

7 Morrow, J., "Cyclic Plastic Strain Energy and Fatigue of Metals," *Internal Friction, Damping and Cyclic Plasticity*, STP 378, 1965, American Society for Testing and Materials, pp. 45–84.

8 Valluri, S. R., "A Unified Engineering Theory of High Stress Level Fatigue," *Aerospace Engineering*, Vol. 20, Oct. 1961, pp. 18–19, 68–69.

9 Weibull, W., *Fatigue Testing and Analysis of Results*, Pergamon Press, New York, 1961, p. 174.

10 Goodman, J., *Mechanics Applied to Engineering*, Longmans, Green and Co., London, 1899.

11 American Society for Testing and Materials Committee E-9, *Manual on Fatigue Testing*, STP 91, American Society for Testing and Materials, 1949.

12 Bockrath, G. E. and Christensen, R. H., "Master Fatigue Curves," Engineering Paper 999, Douglas Aircraft Corp., Long Beach, Calif., 1964.

13 Grover, H. J., Bishop, S. M., and Jackson, L. R., "Fatigue Strengths of Aircraft Materials, Axial-Load Fatigue Tests on Unnotched Sheet Specimens of 24S-T3 and 75S-T6 Aluminum Alloys and of SAE 4130 Steel," TN 2324, 1951, NACA.

14 Gerber, W., "Bestimmung der Zulaessigen Spannungen in Eisen Constructionen," *Zeitschrift des Bayerischen Architekten und Ingenieur Verein*, Vol. 6, 1874, p. 101.

15 Sines, G. and Waisman, J. L., *Metal Fatigue*, McGraw-Hill, New York, 1959, p. 145.

16 Figge, I. E., "An Empirical Equation Relating Fatigue Limit and Mean Stress," TN D-3883, 1967, NASA.

17 Gough, H. J., "Some Experiments on the Resistance of Metals to Fatigue Under Combined Stresses," R and M 2522,

1951, Great Britian Aeronautical Research Council, Ministry of Supply, London, England.

[18] Marin, J., "Interpretation of Fatigue Strengths for Combined Stresses," *Proceedings of International Conference on Fatigue of Metals*, American Society of Mechanical Engineers, 1956, pp. 184–194.

[19] Findley, W. N. and Mathur, P. N., "Modified Theories of Fatigue Failure Under Combined Stress," *Proceedings of Society for Experimental Stress Analysis*, Vol. 14, No. 1, 1956, pp. 35–46.

[20] McClaren, S. W. and Best, J. H., "Low Cycle Fatigue Design Data on Materials in a Multi-Axial Stress Field," RTD-TDR-63-4094, Nov. 1963, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio.

[21] Leybold, H. A., Hardrath, H. F., and Moore, R. L., "An Investigation of the Effects of Atmospheric Corrosion on the Fatigue Life of Aluminum Alloys," TN 4331, 1958, NACA.

[22] Leybold, H. A., "The Effects of Combined Prior Stress and Atmospheric Corrosion on the Fatigue Life of Aluminum Alloys," TN D-2359, 1964, NASA.

[23] Phillips, E. P., "Effect of Outdoor Exposure at Elevated Temperature on the Fatigue Life of Ti-8Al-1Mo-1V Titanium Alloy and AM 350 Stainless Steel Sheet," TN D-5362, 1969, NASA.

[24] Schijve, J. and DeRijk, P., "The Crack Propagation in Two Aluminum Alloys in an Indoor and an Outdoor Environment Under Random and Programmed Load Sequences," NLR-TR M. 2156, National Aerospace Laboratory (Amsterdam), Nov. 1965.

[25] Holshouser, W. L. and Utech, H. P., "Effect of Oleophobic Films on Fatigue Crack Propagation," *Proceedings of the American Society for Testing and Materials*, Vol. 61, 1961, pp. 749–754.

[26] Dunsby, J. A. and Wiebe, W., "Effect of Atmospheric Humidity on Aircraft Structural Alloy Fatigue Life," *Materials Research and Standards*, Vol. 9, No. 2, Feb. 1969, pp. 15–22.

[27] Hardrath, H. F., "Environmental Effects on Fatigue of Structural Materials for a Supersonic Transport," *Advanced Testing Techniques*, STP 476, 1970, American Society for Testing and Materials.

[28] Freudenthal, A. M., "Aspects of Fatigue Damage Accumulation at Elevated Temperatures," *Proceedings of the International Conference on Mechanisms of Fatigue in Crystalline Solids*, Nov. 1962, Orlando, Fla.; also *Acta Metallurgica*, Vol. 11, July 1963.

[29] Spera, D. A., "Calculation of Thermal-Fatigue Life Based on Accumulated Creep Damage," TN D-5489, 1969, NASA.

[30] Coffin, L. F., Chairman, "Fatigue at High Temperature," American STP 459, 1969, American Society for Testing and Materials.

[31] Illg, W., "Fatigue Tests on Notched and Unnotched Sheet Specimens of 2024-T3 and 7075-T6 Aluminum Alloys and of SAE 4130 Steel With Special Consideration of the Life Range From 2 to 10,000 Cycles," TN 3866, 1956, NASA.

[32] Peterson, R. E., *Stress Concentration Design Factors*, Wiley, New York, 1953.

[33] Kuhn P. and Hardrath, H. F., "An Engineering Method for Estimating Notch-Size Effect in Fatigue Tests on Steels," TN 2805, 1952, NACA.

[34] Kuguel, R., "A Relation Between Theoretical Stress Concentration Factor and Fatigue Notch Factor Deduced From the Concept of Highly Stressed Volume," *Proceedings of American Society for Testing and Materials*, Vol. 61, 1961, pp. 732–748.

[35] Griffith, G. E., "Experimental Investigation of the Effects of Plastic Flow in a Tension Panel with a Circular Hole," TN 1705, 1948, NACA.

[36] Stowell, E. Z., "Stress and Strain Concentration at a Circular Hole in an Infinite Plate," TN 2073, 1950, NACA.

[37] Box, W. A., "The Effect of Plastic Strains on Stress Concentrations," *Proceedings of Society for Experimental Stress Analysis*, Vol. 8, No. 2, 1951, pp. 99–110.

[38] Hardrath, H. F. and Ohman, L., "A Study of Elastic and Plastic Stress Concentration Factors Due to Notches and Fillets in Flat Plates," TR 1117, 1953, NASA.

[39] Pian, T. H. H. and D'Amato, R., "Low Cycle Fatigue of Notched and Unnotched Specimens of 2024 Aluminum Alloy Under Axial Loading," TN-58-27, 1958, Wright Aeronautical Development Center, Wright-Patterson Air Force Base, Ohio.

[40] Blatherwick, A. D. and Olson, B. K., "Stress Redistribution in Notched Specimens Under Cyclic Stress," TR-61-451, 1961, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio.

[41] Dixon, J. R., "Elastic-Plastic Strain Distribution in Flat Bars Containing Holes or Notches," *Journal of Mechanics and Physics of Solids*, Vol. 10, 1962, pp. 253–263.

[42] Crews, J. H. and Hardrath, H. F., "A Study of Cyclic Plastic Stresses at a Notch Root," *Journal of Society for Experimental Stress Analysis*, Vol. 6, No. 6, 1966, pp. 313–320.

[43] Crews, J. H., "Elastoplastic Stress-Strain Behavior at Notch Roots in Sheet Specimens Under Constant-Amplitude Loading," TN D-5253, 1969, NASA.

[44] Neuber, H., "Theory of Stress Concentration for Shear-Strained Prismatical Bodies With Arbitrary Nonlinear Stress Strain Law," *Journal of Applied Mechanics*, Vol. 28, Ser. E, No. 4, 1961, pp. 544–550.

[45] Topper, T. H., Wetzel, R. M., and Morrow, J., "Neuber's Rule Applied to Fatigue of Notched Specimens," *Journal of Metals*, Vol. 4, No. 1, March 1969, pp. 200–209.

[46] Holt, M., "Results of Shear Fatigue Tests of Joint with $\frac{3}{16}$ Inch-Diameter 24S-T31 Rivets in 0.064-Inch-Thick Alclad Sheet," TN 2012, 1950, NACA.

[47] Hartmann, E. C., Holt, M., and Eaton, I. D., "Static and Fatigue Strength of High-Strength Aluminum-Alloy Bolted Joints," TN 2276, 1951, NACA.

[48] Schijve, J. and Jacobs, F. A., "Research on Cumulative Damage in Fatigue of Riveted Aluminum Alloy Joints," RM 1999, 1956, National Aerospace Laboratory, Amsterdam.

[49] Schijve, J. and Jacobs, F. A., "The Fatigue Strength of Aluminum Alloy Lugs," TNM2024, 1957, National Aerospace Laboratory, Amsterdam.

[50] Mittenbergs, A. A., "Empirical Analysis of Fatigue Strength of Pin-Loaded Lug Joints, Fatigue Tests of Aircraft Structures: Low-Cycle, Full-Scale and Helicopters," STP 338, 1962, pp. 131–149, American Society for Testing and Materials.

[51] Larsson, S. E., "The Development of a Calculation Method for the Fatigue Strength of Lugs and a Study of Test Results for Lugs of Aluminum," *Fatigue Design Procedures*, edited by E. Gassner and W. Schütz, Pergamon Press, New York, 1965, pp. 309–342.

[52] Raithby, K. D., "Fatigue Tests on Typical Two Spar Light Alloy Structures (Meteor 4 Tailplanes) Under Reversed Loading," Rept. Structures 108, May 1951, British Royal Aeronautical Establishment, Farnborough, England.

[53] McGuigan, M. J., Jr., Bryan, D. F., and Whaley, R. E., "Fatigue Investigation of Full-Scale Transport-Airplane Wings," TN 3190, 1954, NACA.

[54] Whaley, R. E., "Fatigue Investigation of Full-Scale Transport-Airplane Wings—Variable-Amplitude Tests with a Gust-Loads Spectrum," TN 4132, 1957, NACA.

[55] Huston, W. B., "Comparison of Constant-Level and Randomized-Step Tests of Full-Scale Structures as Indicator of Fatigue-Critical Components," *Full-Scale Fatigue Testing of Aircraft Structures*, edited by F. J. Plantema and J. Schijve, Pergamon Press, New York, 1961, pp. 133–148.

[56] Payne, A. O., "Determination of the Fatigue Resistance of Aircraft Wings by Full-Scale Testing," *Full-Scale Fatigue Testing of Aircraft Structures* edited by F. J. Plantema and J. Schijve, Pergamon Press, New York, 1961, pp. 76–132.

[57] Castle, C. B. and Ward, J. F., "Fatigue Investigation of Full-Scale Wing Panels of 7075 Aluminum Alloy," TN D-635, 1961, NASA.

[58] Winkworth, W. J., "Fatigue Behavior Under Service and Ground Test Conditions (A Comparison Based on the Dakota Wing)," Aeronautical Research Council, C.P. No. 666, 1964, Her Majesty's Stationery Office, London, England.

[59] Myers, R. H. and Rosenfeld, M. S., "Variable Amplitude Fatigue Characteristics of the Wing of a Typical Fighter Airplane," Rept. NAEC-ASL-1087, June 1965, U.S. Naval Air Engineering Center, Warminster, Pa.

[60] Branger, J., "The Full Scale Fatigue Test on the DH-112 Venom Aircraft," Second Seminar on Fatigue and Fatigue Design, TR 5, New York, June 1964, Columbia University, New York.

[61] Grover, H. T. et al., "Axial-Load Fatigue Properties of 24S-T and 75S-T Aluminum Alloys as Determined in Several Laboratories," Rept. 1190, 1954, NACA.

[62] Grover, H. J., Bishop, S. M., and Jackson, L. R., "Fatigue Strengths of Aircraft Materials—Axial-Load Fatigue Tests on Notched Sheet Specimens of 24S-T3 and 75S-T6 Aluminum Alloys and of SAE 4130 Steel With Stress-Concentration Factors 2.0 and 4.0," TN 2389, 1951, NACA.

[63] Grover, H. J., Hyler, W. S., and Jackson, L. R., "Fatigue Strengths of Aircraft Materials—Axial-Load Fatigue Tests on

Notched Sheet Specimens of 24S-T3 and 75S-T6 Aluminum Alloys and of SAE 4130 Steel With Stress-Concentration Factor of 1.5," TN 2639, 1952, NACA.

[64] Grover, H. J., Bishop, S. M., and Jackson, L. R., "Fatigue Strengths of Aircraft Materials—Axial-Load Fatigue Tests on Notched Sheet Specimens of 24S-T3 and 75S-T6 Aluminum Alloys and of SAE 4130 Steel With Stress-Concentration Factor of 5.0," TN 2390, 1951, NACA.

[65] Jarfall, E. L., "Optimum Design of Joints: The Stress Severity Factor Concept," Paper presented at 5th International Committee on Aeronautical Fatigue Symposium: Aircraft Fatigue-Design, Operational and Economic Aspects, May 1967, Melbourne, Australia.

[66] Smith, C. R., *Tips on Fatigue*, prepared for Bureau of Naval Weapons, Dept. of the Navy, NAVWEPS 00-25-559, 1963, Washington, D.C.

[67] Walker, W. G. and Copp, M. R., "Summary of VGH and V-G Data Obtained From Piston-Engine Transport Airplanes from 1947 to 1958," TN D-29, 1959, NASA.

[68] Staff of Langley Airworthiness Branch, "Operational Experiences of Turbine-Powered Commercial Transport Airplanes," TN D-1392, 1962, NASA.

[69] Coleman, T. L., "Trends in Repeated Loads on Transport Airplanes," *Fatigue Design Procedures*, edited by E. Gassner and W. Schütz, Pergamon Press, New York, 1969, pp. 161–190.

[70] "Airplane Strength and Rigidity—Reliability Requirements, Repeated Loads, and Fatigue," Spec. MIL-A-8866, 1960, Department of Defense, Washington, D.C.

[71] "Airplane Strength and Rigidity—Flight Loads," Spec. MIL-A-8861, 1960, Department of Defense, Washington, D.C.

[72] Miner, M. A., "Cumulative Damage in Fatigue," *Journal of Applied Mechanics*, Vol. 12, No. 3, Sept. 1945, pp. A159–A164.

[73] Palmgren, A., "Die Lebensdauer von Kugellagern," *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 68, No. 14, 1924, pp. 339–341.

[74] Richart, F. E. and Newmark, N. M., "An Hypothesis for the Determination of Cumulative Damage in Fatigue," *Proceedings of American Society for Testing and Materials*, Vol. 48, 1948, pp. 767–800.

[75] Shanley, F. R., "A Theory of Fatigue Based on Unbonding During Reversed Slip," Paper P-350, Nov. 1952, Rand Corp., Santa Monica, Calif.

[76] Henry, D. L., "A Theory of Fatigue Damage in Steel," *Transactions of American Society of Mechanical Engineers*, Vol. 77, 1955, p. 913.

[77] Corten, H. T. and Dolan, T. J., "Cumulative Fatigue Damage," *Proceedings of International Conference on Fatigue of Metals*, Institution of Mechanical Engineers and American Society of Mechanical Engineers, New York, Nov. 1956.

[78] Levy, J. C., "Cumulative Damage in Fatigue—A Design Method Based on the S-N Curve," *Journal of Royal Aeronautical Society*, Vol. 61, July 1957, pp. 485–491.

[79] Freudenthal, A. M. and Heller, R. A., "On Stress Interaction in Fatigue and a Cumulative Damage Rule," *Journal of Aeronautical Sciences*, Vol. 26, No. 7, July 1959.

[80] Grover, J. H., "An Observation Concerning the Cycle Ratio in Cumulative Damage," *Paper presented at Symposium on Fatigue of Aircraft Structures*, STP 274, 1960, American Society for Testing and Materials.

[81] Fuller, J. R., "Cumulative Fatigue Damage Due to Variable-Cycle Loading," *Noise Control*, American Institute of Physics for the Acoustical Society of America, July–Aug. 1961.

[82] Manson, S. S., Nachtigall, A. J., and Freche, J. C., "A Proposed New Relation for Cumulative Fatigue Damage in Bending," *Proceedings of American Society for Testing and Materials*, Vol. 61, 1961, pp. 679–703.

[83] Impellizzeri, L. F., "Cumulative Damage Analysis in Structural Fatigue," *Effects of Environment and Complex Load History on Fatigue Life*, American Society for Testing and Materials STP 462, 1970, pp. 40–68.

[84] Naumann, E. C., Hardrath, H. F., and Guthrie, D. E., "Axial-Load Fatigue Tests of 2024-T3 and 7075-T6 Aluminum-Alloy Sheet Specimens Under Constant- and Variable-Amplitude Loads," TN D-212, 1959, NASA.

[85] Naumann, E. C. and Schott, R. L., "Axial-Load Fatigue Tests Using Loading Schedules Based on Maneuver-Load Statistics," TN D-1253, 1962, NASA.

[86] Naumann, E. C., "Evaluation of the Influence of Load Randomization and of Ground-Air-Ground Cycles on Fatigue Life," TN D-1584, 1964, NASA.

[87] Corbin, P. L. and Naumann, E. C., "Influence of Programming Techniques and of Varying Limit Load Factors on Maneuver Load Fatigue Test Results," TN D-3149, 1966, NASA.

[88] Jacoby, G. H., "Comparison of Fatigue Lives Under Conventional Program Loading and Digital Random Loading," STP 462, Jan. 1970, American Society for Testing and Materials.

[89] Gassner, E. and Horstmann, K. F., "Einfluss Des Start-Lande-Lastwechsels Auf Die Lebensdauer Der Böenbeanspruchten Flügel Von Verkehrsflugzeugen," *Proceedings of Second International Congress in the Aeronautical Sciences*, Pergamon Press, New York, 1962, pp. 763–780.

[90] Schijve, J., "Endurance Under Program-Fatigue Testing," *Full-Scale Fatigue Testing of Aircraft Structures*, edited by F. J. Plantema and J. Schijve, Pergamon Press, New York, 1961, pp. 41–58.

[91] Breyan, W., "Effects of Block Size, Stress Level, and Loading Sequence on Fatigue Characteristics of Aluminum Alloy Box Beams," STP 462, Jan. 1970, American Society for Testing and Materials.

[92] Lundberg, Bo K. O., "Fatigue Life of Airplane Structures," *Journal of Aerospace Sciences*, Vol. 22, No. 6, June 1955, pp. 349–413.

[93] Lundberg, Bo K. O., "Notes on the Level of Safety and the Repair Rate With Regard to Fatigue in Civil Aircraft Structures," TN HE-794, Sept. 1958, The Aeronautical Research Institute of Sweden (FFA), Stockholm, Sweden.

[94] Lundberg, Bo K. O. and Eggwertz, S., "A Statistical Method for Fail-Safe Design with Respect to Aircraft Fatigue," paper presented at Second International Congress of the International Council of the Aeronautical Sciences, Zurich, Switzerland, Sept. 1960.

[95] Freudenthal, A. M., "Reliability Analysis Based on Time to the First Failure," paper presented at Fifth International Committee on Aeronautical Fatigue Symposium, "Aircraft Fatigue—Design, Operational, and Economic Aspects," Melbourne, Australia, May 1967.

[96] Butler, J. P., "The Materials Section and Structural Development Process for Aircraft Structural Integrity Under Fatigue Conditions," paper presented at Conference on Fatigue and Fracture of Aircraft Structures and Materials, Miami Beach, Fla, Dec. 1969.

[97] Irwin, G. R. and Kies, J. A., "Fracturing and Fracture Dynamics" *Welding Journal*, Research Supplement, Feb. 1962, pp. 95s–100s.

[98] Crichlow, W. J., "The Ultimate Strength of Damaged Structures—Analysis Methods with Correlating Test Data," *Proceedings of Symposium on Full-Scale Fatigue Testing of Aircraft Structures*, Amsterdam, Netherlands, June 1959; also Pergamon Press, New York, 1960.

[99] Christensen, R. H. and Denke, P. H., "Crack Strength and Crack Propagation Characteristics of High Strength Metals," ASD TR-61-207, May 1961, Douglas Aircraft Co., Long Beach, Calif.

[100] Bockrath, G. E., and Glassco, J. B., "Fracture Toughness of High Strength Sheet Metals," Engineering Paper No. 1607, March 1963, Douglas Aircraft Co., Long Beach, Calif.

[101] Welbourne, E. R., "An Analysis of Fatigue Crack Propagation in Sheet Material, Using Familiar Concepts," *Proceedings of Symposium on Current Aeronautical Fatigue Problems*, April 1963, Rome, Italy, Pergamon Press, New York, 1965, pp. 265–286.

[102] Broek, D., "The Residual Strength of Light Alloy Sheets Containing Fatigue Cracks," M.P. 241, 1966, National Aerospace Laboratory, Amsterdam,

[103] Hahn, G. T. and Rosenfield, A. R., "Sources of Fracture Toughness: The Relation Between K_{Ic} and the Ordinary Tensile Properties of Metals," (Applications Related Phenomena in Titanium Alloys), STP 432, 1968, American Society for Testing and Materials.

[104] Kuhn, P., "Residual Strength in the Presence of Fatigue Cracks," paper presented to the *Structures and Materials Panel—Advisory Group for Aeronautical Research and Development*, Secs. 1–4 (Turin, Italy), April 1967, Secs. 5–7 (Ottawa, Canada) Sept. 1967.

[105] Irwin, G. R., "Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate," *Transactions of American Society for Mechanical Engineers*, Vol. 24, No. 3, 1957, pp. 361–364.

[106] Paris, P. C. and Sih, G. C., "Stress Analysis of Cracks, Fracture Toughness Testing and Its Applications," STP 381, 1965, American Society for Testing and Materials, pp. 30–83.

[107] Brown, W. F., Jr. and Srawley, J. G., "Plane Strain Crack Toughness Testing of High Strength Metallic Materials," STP 410, 1966, American Society for Testing and Materials.
[108] Allen, F. C., "The Stress Analysis of Centrally Cracked Plates," Engineering Paper No. 5513, March 26, 1969, Douglas Aircraft Co., Long Beach, Calif.
[109] American Society for Testing and Materials, "Proposed Recommended Practice for Plane-Strain Fracture Toughness Testing of High-Strength Metallic Materials Using a Fatigue-Crack Bend Specimen," *American Society for Testing and Materials Standards*, Part 31, May 1968, p. 1018.
[110] Dugdale, D. S., "Yielding of Steel Sheets Containing Slits," *Journal of Mechanics and Physics of Solids*, Vol. 8, 1960.
[111] Barenblatt, G. I., "Mathematical Theory of Equilibrium Cracks in Brittle Fracture," *Advances in Applied Mechanics*, Vol. VII, Academic Press, New York, 1962.
[112] Mukherjee, A. K. et al., "Notch Behavior in Metals," AFML-TR-66-266, 1966, Wright-Patterson Air Force Base, Ohio.
[113] Swedlow, J. L., "Elasto-Plastic Cracked Plates in Plane Strain," *International Journal of Fracture Mechanics*, Vol. 5, No. 1, March 1969, pp. 33–44.
[114] Newman, J. C., Jr., "Fracture of Cracked Plates Under Plane Stress," *International Journal of Engineering Fracture Mechanics*, Vol. I, No. 1, 1968.
[115] Rice, J. R., and Rosengren, G. F., "Plane Strain Deformation Near a Crack Tip in a Power-Law Hardening Material," *Journal of Mechanics and Physics of Solids*, Vol. 16, No. 1, Jan. 1968, pp. 1–12.
[116] Hutchinson, J. W., "Singular Behavior at the End of a Tensile Crack in a Hardening Material," *Journal of Mechanics and Physics of Solids*, Vol. 16, No. 1, Jan. 1968, pp. 13–31.
[117] Hilton, P. D. and Hutchinson, J. W., "Plastic Intensity Factors for Cracked Plates," Rept. SM-34, May 1969, Harvard Univ. (accepted for publication in the International Journal of Engineering Fracture Mechanics).
[118] Irwin, G. R., "Fracture," *Handbuch der Physik*, Vol. VI, Springer, Berlin, 1958.
[119] Bowie, O. L., "Analysis of an Infinite Plate Containing Radial Cracks Originating from the Boundary of an Internal Circular Hole," *Journal of Mathematics and Physics*, Vol. 35, No. 1, 1956, pp. 60–71.
[120] Romauldi, J. P. and Sanders, P. H., "Fracture Arrest by Riveted Stiffeners," AFOSR TR 60-174, Oct. 1960, Air Force Office of Scientific Research.
[121] Figge, I. E. and Newman, J. C., Jr., "Fatigue Crack Propagation in Structures with Simulated Rivet Forces," STP 415, 1967, American Society for Testing and Materials Standards, p. 71.
[122] Poe, C. C., Jr., "The Effect of Riveted and Uniformly Spaced Stringers on the Stress Intensity Factor of a Cracked Sheet," paper presented at Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials (Miami Beach, Fla.), Dec. 1969.
[123] Smith, S. H., Porter, T. R., and Engstrom, W. L., "Fatigue Crack Propagation Behavior and Residual Strength of Bonded Strap Reinforced, Laminated and Sandwich Panels," paper presented at Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials (Miami Beach, Fla.), Dec. 1969.
[124] Williams, M. L., Ed., *International Journal of Fracture Mechanics*, Wolters-Noordhoff Publishing, Groningen, The Netherlands.
[125] Irwin, G. R., Liebowitz, H., and Paris, P. C., Eds., *International Journal of Engineering Fracture Mechanics*, Pergamon Press, New York.
[126] American Society for Testing and Materials, "Fracture Toughness and Its Applications," ASTM STP 381, April 1965.
[127] American Society for Testing and Materials, "Plane Strain Crack Toughness Testing of High Strength Metallic Materials," ASTM STP 410, Dec. 1967.
[128] American Society for Testing and Materials, "Fatigue Crack Propagation," ASTM STP 415, June 1967.
[129] Yokobori, T., Kawasaki, T., and Swedlow, J. L., Eds., *Proceedings of the First International Conference on Fracture (Sendai)*, The Japanese Society for Strength and Fracture of Materials, c. 1966 (Vol. I—Mathematical, Physical and Continuum Mechanical Theories; Vol. II—Atomistic, Microstructural and Macroscopic Mechanics; Vol. III—Strength and Fracture of Non-Metallic Materials).
[130] Pratt, P. L., Ed. "Fracture 1969," *Proceedings of the Second International Conference on Fracture (Brighton)*, Chapman and Hall Ltd., London, 1969.
[131] Anderson, W. E., "Application of Fracture Toughness Considerations to Material Selection and Design," lecture presented to Pittsburgh Chapter, American Society for Metals, 1965.
[132] Piper, D. E., Quist, W. E., and Anderson, W. E., "The Effect of Composition on the Fracture Properties of 7178-T6 Aluminum Alloy Sheet," *Application of Fracture Toughness Parameters to Structural Metals*, Vol. 31, Gordon and Breach Science Publishers, Inc., New York, 1966, pp. 227–280.
[133] Hyatt, M. V. and Quist, W. E., "The Effect of Zinc/Magnesium Ratio and Copper Content on the Fatigue Properties of Al-Zn-Mg-Cu Alloys," paper presented at American Society for Testing and Materials Symposium on Achievement of High Fatigue Resistance in Metals and Alloys (Atlantic City, N. J.), June 1969.
[134] Ostermann, F. G. and Reimann, W. H., "Thermo-Mechanical Processing and Fatigue of Aluminum Alloys," STP 467, 1970, American Society for Testing and Materials.
[135] Irwin, G. R., "Fracture Mode Transition for a Crack Traversing a Plate," *Transaction of American Society for Mechanical Engineers*, No. 417, June 1960.
[136] Head, A. K., "The Growth of Fatigue Cracks," *Philosophical Magazine*, Ser. 7, Vol. 44, No. 356, Sept. 1953, pp. 925–938.
[137] Weibull, W., "The Effect of Size and Stress History on Fatigue Crack Initiation and Propagation," *Proceedings of the Crack Propagation Symposium*, The College of Aeronautics, Cranfield, England, 1962, pp. 271–286.
[138] Frost, N. E. and Dugdale, D. S., "The Propagation of Fatigue Cracks in Sheet Specimens," *Journal of Mechanics and Physics of Solids*, Vol. 6, No. 2, 1958.
[139] Illg, W. and McEvily, A. J., Jr., "The Rate of Fatigue-Crack Propagation for Two Aluminum Alloys Under Completely Reversed Loading," TN D-52, 1959, NASA.
[140] Liu, H. W., "Fatigue Crack Propagation and Applied Stress—An Energy Approach," *Transactions of the American Society of Mechanical Engineers, Ser. D: Journal of Basic Engineering*, Vol. 85, 1963, p. 116.
[141] Paris, P. and Erdogan, F., "A Critical Analysis of Crack Propagation Laws," *Transactions of the American Society of Mechanical Engineers*, Ser. D: *Journal of Basic Engineering*, Vol. 85, Dec. 1963, pp. 528–534.
[142] Paris, P. C., "The Fracture Mechanics Approach to Fatigue," *Fatigue—An Interdisciplinary Approach*, edited by Burke, J. J., Reed, N. L., and Weiss, V., Syracuse University Press, 1964, pp. 107–132.
[143] *Proceedings of the Crack Propagation Symposium*, The College of Aeronautics, Cranfield, England, 1962.
[144] Broek, D. and Shijve, J., "The Influence of the Mean Stress on the Propagation of Fatigue Cracks in Aluminum Alloy Sheet," NLR-TR m. 2111, 1963, National Aerospace Laboratory, Amsterdam.
[145] Erdogan, F., "Crack Propagation Theories," CR-901, 1967, NASA.
[146] Forman, R. G., Kearney, V. E., and Engle, R. M., "Numerical Analysis of Crack Propagation in Cyclic-Loaded Structures," *Transactions of the American Society of Mechanical Engineers, Ser. D: Journal of Basic Engineering*, Vol. 89, No. 3, Sept. 1967, pp. 459–464.
[147] Hudson, C. M., "Effect of Stress Ratio on Fatigue-Crack Growth in 7075-T6 and 2024-T3 Aluminum-Alloy Specimens," TN D-5390, Aug. 1969, NASA.
[148] Hudson, C. M. and Hardrath, H. F., "Effects of Changing Stress Amplitude on the Rate of Fatigue-Crack Propagation in Two Aluminum Alloys," TN D-960, 1961, NASA.
[149] Schijve, J., Broek, D., and DeRijk, P., "Fatigue-Crack Propagation Under Variable-Amplitude Loading," NLR-TN M. 2094, 1961, National Aerospace Laboratory, Amsterdam.
[150] Hudson, C. M. and Hardrath, H. F., "Investigation of the Effects of Variable-Amplitude Loadings on Fatigue Crack Propagation Patterns," TN D-1803, 1963, NASA.
[151] Christensen, R. H., "Prediction of Cumulative Fatigue-Crack Growth Under Complex Environments," Engineering Paper No. 1461, Oct. 1962, Douglas Aircraft Co., Long Beach, Calif.
[152] Schijve, J., "Fatigue Life and Crack Propagation under Random and Programmed Load Sequences," *Current Aeronautical Fatigue Problems*, Pergamon Press, New York, 1965, p. 403.
[153] McMillan, J. C. and Pelloux, R. M. N., "Fatigue Crack

Propagation Under Programmed and Random Loads," D1-82-0553, July 1966, Boeing Scientific Research Laboratories, Seattle, Wash.

154 Hertzberg, R. W., "Fatigue Fracture Surface Appearance," STP 415, June 1967, American Society for Testing and Materials.

155 Hudson, C. M. and Raju, K. N., "Investigation of Fatigue Crack Growth Under Simple Variable-Amplitude Loading," TN D-5702, 1970, NASA.

156 Swanson, S. R., Cicci, F., and Hoppe, E., "Crack Propagation in Clad 7079-T6 Aluminum Alloy Sheet Under Constant and Random Amplitude Fatigue Loading," STP 415, June 1967, American Society for Testing and Materials.

157 Tiffany, C. F. and Master, J. N., "Applied Fracture Mechanics, Fracture Toughness Testing and Its Applications," STP 381, 1965, American Society for Testing and Materials, pp. 249–278.

158 Beachem, C. D. and Brown, B. F., "A Comparison of Three Precracked Specimens for Evaluating the Susceptibility of High-Strength Steel to Stress Corrosion Cracking," STP 425, 1967, American Society for Testing and Materials.

159 Tiffany, C. F. and Masters, J. N., "Investigation of the Flaw Growth Characteristics of 6 Al-4V Titanium Used in Apollo Spacecraft Pressure Vessels," CR 65586, March 1967, NASA.

160 Donaldson, D. R. and Anderson, W. E., "Crack Propagation Behavior of Some Airframe Materials," *Proceedings of Crack Propagation Symposium*, Vol. II, The College of Aeronautics, Cranfield, England, Sept. 1961.

161 Sanga, R. V. and Porter, T. R., "Application of Fracture Mechanics for Fatigue Life Prediction," paper presented at Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials (Miami Beach, Fla.), Dec. 1969.

Bibliography

Fatigue

Books and Symposia

Murray, W. M., Ed., *Fatigue and Fracture of Metals*, Wiley, New York, 1952.

Grover, H. J. and Jackson, L. R., "Fatigue of Metals and Structures," NAVAER 00-25-534, 1954, Bureau of Aeronautics, Dept. of Navy, Washington, D.C.

The Institution of Mechanical Engineers and The American Society of Mechanical Engineers, *Proceedings of the International Conference on Fatigue of Metals* (London), Sept. 1956 (New York), Nov. 1956.

Sines, G., and Waisman, J. L., Eds., *Metal Fatigue*, McGraw-Hill, New York, 1959.

Pope, J. A., *Metal Fatigue*, Chapman and Hall, London, 1959.

Weibull, W., *Fatigue Testing and Analysis of Results*, Pergamon Press, New York, 1961.

"Mechanisms of Fatigue in Crystalline Solids," *Proceedings of an International Conference on the Mechanisms of Fatigue in Crystalline Solids* (Orlando, Fla., Nov. 1962), Pergamon Press, New York, c. 1963.

Heywood, R. B., *Designing Against Fatigue of Metals*, Reinhold, New York, 1962.

Kennedy, A. J., *Processes of Creep and Fatigue in Metals*, Oliver and Boyd, Edinburgh and London, 1962.

Burke, J. J., Reed, N. L., and Weiss, V., Eds., "Fatigue—An Interdisciplinary Approach," *Proceedings of the 10th Sagamore Army Materials Research Conference* (Raquette Lake, N. Y.), Aug. 1963, Syracuse University Press, 1964.

Trapp, W. J. and Forney, D. M., Jr., "Acoustical Fatigue in Aerospace Structures," *Proceedings of the Second International Conference* (Dayton, Ohio), April–May, 1964, Syracuse University Press, c. 1965.

Yokobori, T., *The Strength, Fracture, and Fatigue of Materials*, P. Noordhoff, Groningen, The Netherlands, 1965.

Grover, H. J., "Fatigue of Aircraft Structures," NAVAIR 01-1A-13, Naval Air Systems Command, Dept. of Navy, 1966.

Mann, J. Y., *Fatigue of Materials*, Melbourne University Press, c. 1967.

Graham, J. A., Ed., *Fatigue Design Handbook*, Society of Automotive Engineers, Inc., New York, 1968,

Madayag, A. F., Ed., *Metal Fatigue: Theory and Design*, Wiley, New York, c. 1969.

International Committee on Aeronautical Fatigue

Plantema, F. J. and Schijve, J., Eds., "Full-Scale Fatigue Testing of Aircraft Structures," *Proceedings of the Symposium Held in Amsterdam*, June 1959, Vol. 5, Pergamon Press, New York, 1961.

Barrois, W. and Ripley, E. L., Eds., "Fatigue of Aircraft Structures," *Proceedings of the Symposium Held in Paris*, May 1961, Vol. 12, Macmillan, New York, 1963.

Schijve, J., Heath-Smith, J. R., and Welbourne, E. R., Eds., "Current Aeronautical Fatigue Problems," *Proceedings of a Symposium Held in Rome*, April 1963, Vol. 19, Pergamon Press, c. 1965.

Gassner, E. and Schütz, W., Eds., "Fatigue Design Procedures," *Proceedings of the Symposium Held in Munich*, June 1965, Vol. 20, Pergamon Press, c. 1969.

American Society for Testing and Materials

"Fatigue of Aircraft Structures," ASTM STP 203, c. 1957.

"Large Fatigue Testing Machines and Their Results," ASTM STP 216, c. 1958.

"Symposium on Basic Mechanisms of Fatigue," ASTM STP 237, c. 1959.

"Symposium on Fatigue of Aircraft Structures," ASTM STP 274, c. 1960.

"Symposium on Fatigue Tests of Aircraft Structures (Low-Cycle, Full-Scale, and Helicopters)," ASTM STP 338, c. 1963.

"A Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data," ASTM STP 91-A, 2nd ed., c. 1963.

"Structural Fatigue in Aircraft," ASTM STP 404, c. 1966.

"Fatigue at High Temperature," ASTM STP 459, c. 1969.

"Effects of Environment and Complex Load History on Fatigue Life," ASTM STP 462, c. 1970.

"Manual on Low Cycle Fatigue Testing," ASTM STP 465, c. 1969.

"References on Fatigue," ASTM STP 9, 1951–1967.

"Low Cycle Fatigue Bibliographic Survey," ASTM STP 449, c. 1968.

Fracture

Books and Symposia

Jonassen, F., Roop, W. P., and Bayless, R. T., "Fracturing of Metals," *A Seminar Held During the Twenty-Ninth National Metal Congress and Exposition* (Chicago), Oct. 1947, The American Society for Metals, c. 1948.

Proceedings of the Crack Propagation Symposium, 2 Vols., 1961, The College of Aeronautics, Cranfield, England.

Tetelman, A. S., and McEvily, A. J., Jr., *Fracture of Structural Materials*, Wiley, New York, c. 1967.

Yokobori, T., Kawasaki, T., and Swedlow, J. L., Eds., *Proceedings of the First International Conference on Fracture* (Sendai), c. 1966, The Japanese Society for Strength and Fracture of Materials (Vol. I—*Mathematical, Physical, and Continuum Mechanical Theories*; Vol. II—*Atomistic, Microstructural, and Macroscopic Mechanics*; Vol. III—*Strength and Fracture of Non-Metallic Materials*).

Pratt, P. L., Ed., "Fracture 1969," *Proceedings of the Second International Conference on Fracture* (Brighton), April 1969, Chapman and Hall Ltd., London, Eng.

Liebowitz, H., Ed., "Fracture—An Advanced Treatise," Academic Press, New York, 1968 (Vol. I—Microscopic and Macroscopic Fundamentals; Vol. II—Mathematical Fundamentals; Vol. III—Engineering Fundamentals and Environmental Effects; Vol. IV—Engineering Fracture Design; Vol. V—Fracture Design of Structures; Vol. VI—Fracture of Metals; Vol. VII—Fracture of Nonmetals and Composites).

American Society for Testing and Materials

"Fracture Toughness Testing and Its Applications," ASTM STP 381, c. 1965.

"Stress-Corrosion Cracking of Titanium," ASTM STP 397, c. 1966.

"Plane Strain Crack Toughness Testing of High Strength Metallic Materials," ASTM STP 410, c. 1966.

"Fatigue Crack Propagation," ASTM STP 415, c. 1967.

"Stress Corrosion Testing," ASTM STP 425, c. 1967.